Math 10B with Professor Stankova
Worksheet, Discussion #21; Tuesday, 4/10/2018
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1 Miscellaneous

1.1 Concepts

1. **Euler's formula** tells us that $e^{i\theta} = \cos\theta + i\sin\theta$.

1.2 Example

2. Show that $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$.

Solution: We have that the right side is

$$\frac{\cos\theta + i\sin\theta - (\cos(-\theta) + i\sin(-\theta))}{2i} = \frac{\cos\theta + i\sin\theta - (\cos\theta - i\sin\theta)}{2i} = \frac{2i\sin\theta}{2i} = \sin\theta.$$

2 Slope Fields

2.1 Concepts

3. A slope field is a graph where at every point y, t, you draw a line with the slope there, which is given by the function $f(y, t)$.

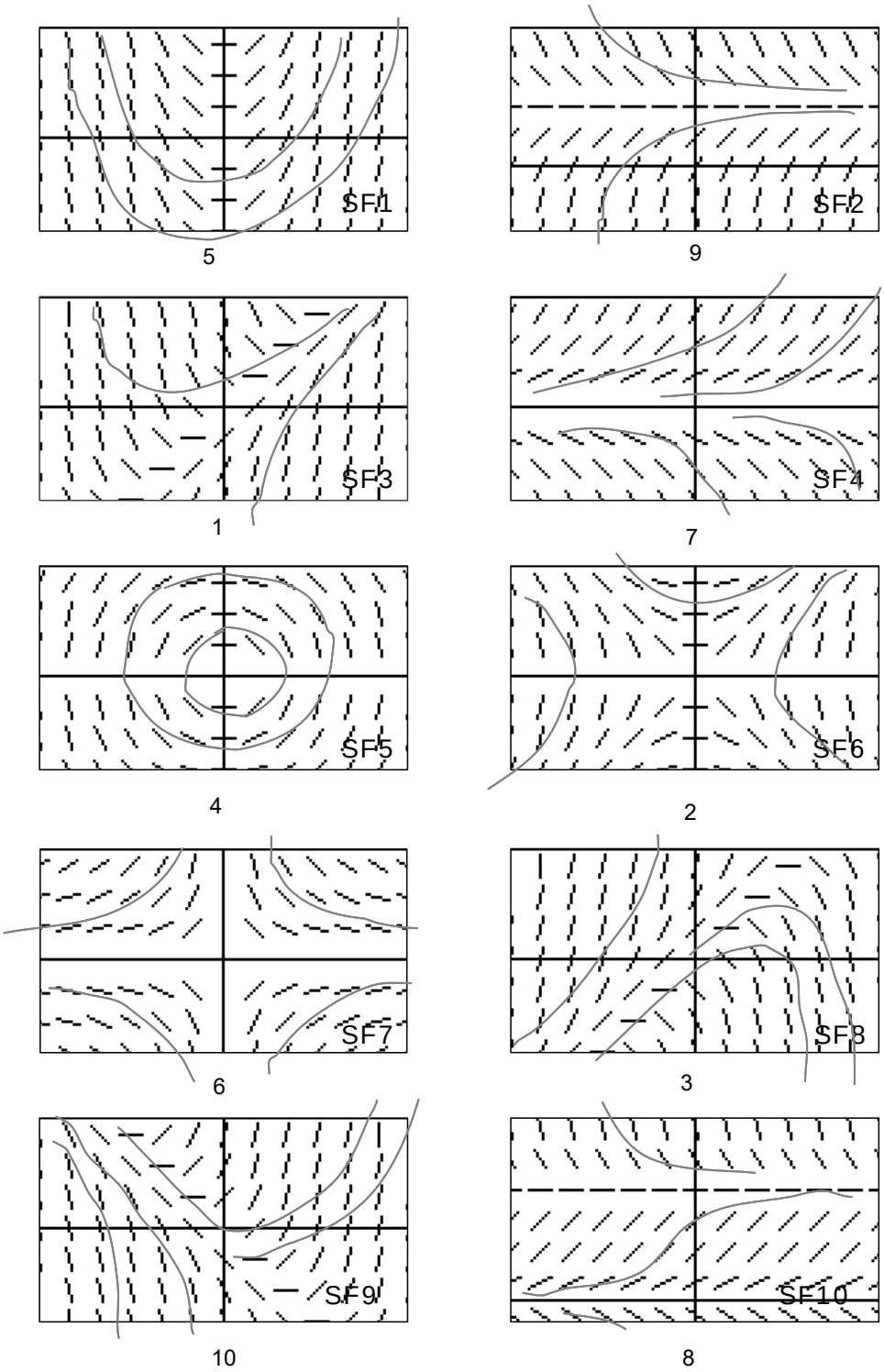
2.2 Problems

4. Match each slope field to the differential equation and sketch some solutions to them.

5. For each differential equation, estimate $y(2)$ using the starting point $y(1) = 1$ and step size of $h = \frac{1}{2}$.

Solution:

DE	$y(1.5)$	$y(2)$
	$y(1)+f(1,y(1))h$	$y(1.5)+f(1.5,y(1.5))h$
1	$1+(1-1)(0.5)=1$	$1+(1.5-1)(0.5)=1.25$
2	$1+(1/1)(0.5)=1.5$	$1.5+(1.5/1.5)(0.5)=2$
3	$1+(1-1)(0.5)=1$	$1+(1-1.5)(0.5)=0.75$
4	$1+(-1/1)(0.5)=0.5$	$0.5+(-1.5/0.5)(0.5)=-1$
5	$1+(1)(0.5)=1.5$	$1.5+(1.5)(0.5)=2.25$
6	$1+(-1/1)(0.5)=0.5$	$0.5+(-0.5/1.5)(0.5)=\frac{1}{3}$
7	$1+(1/2)(0.5)=1.25$	$1.25+(1.25/2)(0.5)=\frac{25}{16}$
8	$1+0.25(1)(4-1)(0.5)=1.75$	$1.75+0.25(1.75)(2.25)=\frac{175}{64}$
9	$1+(2-1)(0.5)=1.5$	$1.5+(2-1.5)(0.5)=1.75$
10	$1+(1+1)(0.5)=2$	$2+(1.5+2)(0.5)=3.75$


SF1
5
SF2
9
SF3
1
SF4
7
SF5
4
SF6
2
SF7
6
SF8
3
SF9
10
SF10
8

$$\frac{dy}{dx} = x - y$$

DE1

$$\frac{dy}{dx} = \frac{x}{y}$$

DE2

$$\frac{dy}{dx} = y - x$$

DE3

$$\frac{dy}{dx} = -\frac{x}{y}$$

DE4

$$\frac{dy}{dx} = x$$

DE5

$$\frac{dy}{dx} = -\frac{y}{x}$$

DE6

$$\frac{dy}{dx} = \frac{y}{2}$$

DE7

$$\frac{dy}{dx} = 0.25y(4 - y)$$

DE8

$$\frac{dy}{dx} = 2 - y$$

DE9

$$\frac{dy}{dx} = x + y$$

DE10